16. Estimation of the Production Cost for a LEGO™ Brick

(Inspired from John Hart, MIT)

The purpose of this exercise is to estimate the cost for producing LEGO bricks.

The following information are provided:

- *Material:* ABS
- *Density:* $\rho_{ABS} = 1000\ \mathrm{kg \cdot m^{-3}}$
- *Raw ABS cost:* $3.50\ \$ \cdot \mathrm{kg}^{-1}$
- *Basic overhead:* $10\ \$ \cdot \mathrm{h}^{-1}$
- *Injection molding cycle time:* $\sim 6\ \mathrm{s}$
- *Mold cost (for 8 bricks):* $\sim 35\ 000\ \$$
- *Machine cost:* $\sim 200\ 000\ \$$ *(with a depreciation time of* 10 *years)*
- *Void volume fraction of a single brick:* 65.6%

Typical dimensions for a LEGO brick are shown in the side figure.

This YouTube video illustrates how LEGO bricks are made: https://youtu.be/y1Zhpdx-XtA.

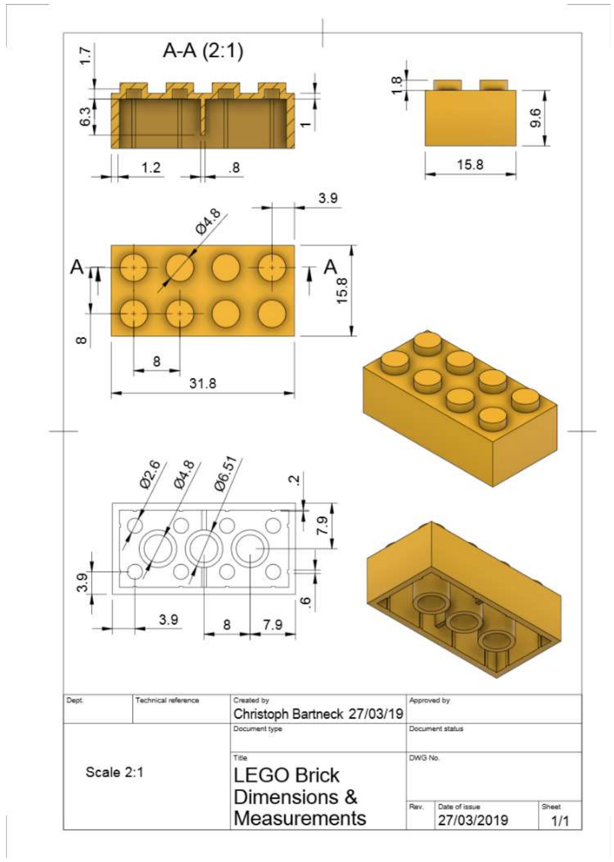


Figure 28. *Typical dimensions of a LEGO brick that can be used to estimate relevant volumetric parameters.*

1. Write the equation for the total shaping cost per part with its four terms (real cost of manufactured material C_{mm}, cost of tooling (or dedicated cost) $C_{tooling}$, capital rate $\dot{C}_{capital}$ and overhead rate $\dot{C}_{oh}$), as a function of the number of parts produced n and production rate $\dot{n}$.

The total shaping cost per part is given by:

$$C = C_{mm} + \frac{\dot{C}_{capital} + \dot{C}_{oh}}{\dot{n}} + \frac{C_{tooling}}{n}$$

This equation shows that the cost has three essential contributions:

- A material cost per unit of prod. that is independent of batch size and production rate
- A gross overhead per unit of prod. that varies as the reciprocal of the production rate $1/\dot{n}$
- A dedicated cost per unit of prod. that varies as the reciprocal of the production volume $1/n$

2. Develop this general equation with the literal relations seen in the lecture on manufacturing economics. Make sure to identify which expression corresponds to which term.

Developing each term by respecting the order followed in the first question, we obtain:

$$C = \left(\frac{m}{1-f}\right)C_{rm} + \frac{1}{\dot{n}}\left[\left(\frac{1}{L}\right)\frac{C_c}{t_{wo}} + \dot{C}_{oh}\right] + \left(\frac{C_t}{n}\right)\left\{1 + E\left(\frac{n}{n_t}\right)\right\}$$

The dedicated cost, the effective hourly rate of capital write-off and the production rate can all be defined by a representative range for each process.

The target batch size n, the overhead rate $\dot{C}_{oh}$, the load factor L and the capital write-off time t_{wo} must be defined by the user.

Let us apply this formula to calculate the overall fabrication cost of a Lego brick in the next question.

3. Calculate the cost of a brick considering a production level of (a) 1000 parts, (b) 100 000 parts, and (c) 10 000 000 parts.

Note that the real cost of manufactured material C_{mm}, the cost of tooling $C_{tooling}$, the equipment cost C_e and the other costs C_{oth} are calculated in dollars/part.

A. Manufactured Material Cost

The cost of material is given by the first term of the cost equation:

$$C_{mm} = \left(\frac{m}{1-f}\right)C_{rm}$$

Fraction of the raw material lost during the process $\quad f = 0\%$
(could be assumed to be 0%, often a value of 1% is used)

Cost of raw material $\quad C_{rm} = 3.50\ \$ \cdot \mathrm{kg}^{-1}$

The volume of the brick is given by the dimensions of the brick:

$$V_{brick} = \left[(9.6 \cdot 31.8 \cdot 15.8) + \left(\frac{4.8}{2}\right)^2 \pi \cdot 1.8 \cdot 8\right] \cdot (1 - 0.656) = 1749\ mm^3 = 1.749 \cdot 10^{-6}\ \mathrm{m}^3$$

Mass of the LEGO brick $\quad m_{brick} = \rho_{ABS} V_{brick}$

$$m_{brick} = 1000 \cdot 1.749 \cdot 10^{-6} = 1.749 \cdot 10^{-3}\ \mathrm{kg}$$

<u>Total manufactured material cost:</u>

$$C_{mm} = m_{brick} \cdot C_{rm} = 1.749 \cdot 10^{-3} \cdot 3.5 \cong 0.006\ \$$$

B. Equipment Cost

The cost of the equipment is given by the second term of the cost equation:

$$C_e = \frac{1}{\dot{n}}\left[\left(\frac{1}{L}\right)\frac{C_c}{t_{wo}} + \dot{C}_{oh}\right]$$

Production rate	$\dot{n} = \frac{8\,parts}{6\,s} = 4800 \text{ parts} \cdot \text{h}^{-1}$
Machine usage *(not given, can be assumed to be 50%)*	$L = 50\%$
Machine cost	$C_c = 200\,000\,\$$
Depreciation time	$t_{wo} = 10\,years = 87\,600 \text{ h}$
Overhead costs	$\dot{C}_{oh} = 10\,\$ \cdot \text{h}^{-1}$

Total equipment cost:

$$C_e = \left(\frac{1}{4800}\right)\left[\left(\frac{1}{0.5}\right)\frac{200\,000}{87\,600} + 10\right] \cong 0.005\,\$$$

C. Dedicated Cost

The cost of tooling is given by the third part of the cost equation:

$$C_{tooling} = \left(\frac{C_t}{n}\right)\left\{1 + E\left(\frac{n}{n_t}\right)\right\}$$

Assuming that the mold never deteriorates, no tool replacement is needed, so $\left\{1 + E\left(\frac{n}{n_t}\right)\right\} = 1.$

Cost of tools	$C_t = 35\,000\,\$$

The following tooling costs $C_{tooling}(n)$ are obtained for different amounts n of fabricated parts:

$$C_{tooling}(1000) = \frac{35\,000}{1000} = 35\,\$$$

$$C_{tooling}(100\,000) = \frac{35\,000}{100\,000} = 0.35\,\$$$

$$C_{tooling}(10\,000\,000) = \frac{35\,000}{10\,000\,000} = 0.0035\,\$$$

Note that it would be tempting to divide by eight the cost, since the mold produces eight parts at a time. However, this is incorrect as the mold cost is a fixed investment. The fact that it produces four, eight or more per batch does not change the cost you have to pay for the investment. However, it has a direct impact on the production rate.

D. Total Cost

The total cost of a single Lego brick is the addition of the manufactured material, equipment and tooling costs. The cost can be calculated for each value of n:

$$C_{1000} = 0.006 + 0.005 + 35 = 35.011\,\$$$

$$C_{100\,000} = 0.006 + 0.005 + 0.35 = 0.361\,\$$$

$$C_{10\,000\,000} = 0.006 + 0.005 + 0.0035 = 0.0145\,\$$$

Note that in case of a product (e.g. in your reverse engineering project), you would need to compute the cost for producing *each* part and add the *assembly cost* (see the lecture on assembly) to estimate the total production cost of your product.

17. Estimation of the Production Cost for Laser Processed Parts

We consider the laser processing of glass made by combining femtosecond lasers and chemical etching. Details about this process are discussed in the lecture on laser manufacturing and in a previous exercise. Our goal is to estimate the cost for producing a part using this advanced manufacturing technology and discuss a way of lowering production costs.

Our working hypothesis are:

- *Material:* fused silica glass
- *Raw material cost:* $\sim 10\ \mathrm{CHF} \cdot \mathrm{substrate}^{-1}$
- *Basic overhead:* $\sim 10\ \mathrm{CHF} \cdot \mathrm{h}^{-1}$
- *Machine cost:* $\sim 400\,000\ \mathrm{CHF}$ *(with a depreciation time of* 5 *years)*
- *Processing time:* $\sim 1\ \mathrm{h} \cdot \mathrm{part}^{-1}$
- *We assume that* 4 *parts per substrate can be produced*

1. What should be the selling price per part for a batch of 1000 parts with a 30% profit margin?

A. Manufactured Material Cost

In this case, the raw material is a fused silica substrate. As 4 parts can be produced with a single substrate, each part costs 25% of the price of a substrate.

With the given price of 10 CHF by substrate, the material cost per part is easily calculated:

$$C_{mm} = 0.25 \cdot 10 = 2.5\ \mathrm{CHF}$$

B. Equipment Cost

The cost of the equipment is given by the second term of the cost equation:

$$C_e = \frac{1}{\dot{n}}\left[\left(\frac{1}{L}\right)\frac{C_c}{t_{wo}} + \dot{C}_{oh}\right]$$

Production rate	$\dot{n} = 1\ \mathrm{part} \cdot \mathrm{h}^{-1}$
Machine usage *(not given, can be assumed to be 50%)*	$L = 50\%$
Machine cost	$C_c = 400\,000\ \mathrm{CHF}$
Depreciation time	$t_{wo} = 5\ years = 43\,800\ \mathrm{h}$
Overhead costs	$\dot{C}_{oh} = 10\ \mathrm{CHF} \cdot \mathrm{h}^{-1}$

<u>Total equipment cost:</u>

$$C_e = \left(\frac{1}{1}\right)\left[\left(\frac{1}{0.5}\right)\frac{400\,000}{43\,800} + 10\right] \cong 28.3\ \mathrm{CHF}$$

C. Dedicated Cost

No specific tools are required in this process: $C_{tooling} = 0$.

D. Total Cost

The total cost is obtained by adding the different costs:

$$C = 2.5 + 28.3 = 30.8 \text{ CHF}$$

The profit margin is defined as the ratio between the net profit and the selling price:

$$profit\ \ margin = \frac{revenue - cost}{revenue}$$

Therefore, the price per part (or revenue) if a 30% profit margin is desired is:

$$P = \frac{cost}{1 - 0.3} = \frac{30.8}{0.7} \cong 44 \text{ CHF}$$

Two remarks:

- The final price is independent on the number of parts that are produced!
- This price is probably underestimated as we did not consider the costs related to the etching part of the process (etching solution, fume hood, disposal facilities, ...). However, it is possible that these costs be nearly neglectable with respect to the laser costs.

2. How could we reduce the production cost?

Given the input parameters and assuming that the machine usage cannot be increased (because of regulations), increasing the speed of the process could be a solution to lower the cost.

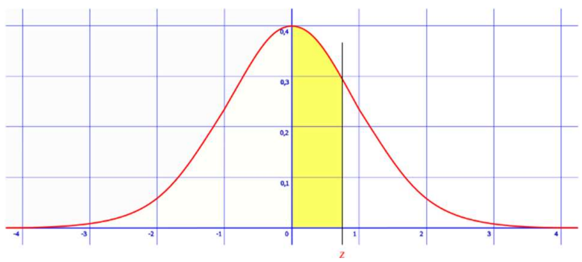
18. Brush-Up on Statistics

Let us consider a drill bit and do the following assumptions:

- The drill bit's lifetime can be modelled by a normal distribution
- It has an average lifetime of 5000 holes
- It has a standard deviation of 500 holes

1. What fraction of drill bits are likely to wear out after (a) 4500 holes? (b) 5500 holes? (c) 6000 holes? Please use the z-scores table provided on the next page.

At first, since the distribution follows a Gaussian curve centred at a mean of $\mu = 5000$ holes with a standard deviation of $\sigma = 500$ holes, we can use the z-score table to define the fraction of drill bits that will wear out after a drilling a certain number of holes.



The z-score is defined as follows, where x is the variable, μ the mean and σ the standard deviation:

$$z = \frac{x - \mu}{\sigma}$$

Hence, we have:

$$P(x \leq 4500) = P\left(z \leq \frac{4500 - 5000}{500}\right) = P(z \leq -1) = P(z \geq 1) = 1 - (0.5 + 0.34134) = 15.87\%$$

$$P(x \leq 5500) = P\left(z \leq \frac{5500 - 5000}{500}\right) = P(z \leq 1) = 0.5 + 0.34134 = 84.13\%$$

$$P(x \leq 6000) = P\left(z \leq \frac{6000 - 5000}{500}\right) = P(z \leq 2) = 0.5 + 0.47725 = 97.72\%$$

a. According to the table, 15.87% of the pieces are expected to wear out after drilling 4500 holes.
b. 84.13% of the drill bits should be replaced after drilling 5500 holes.
c. 97.72% of the parts are expected to wear out after drilling 6000 holes.

2. What fraction of drill bits will wear out at exactly the 5725th hole?

Remember that we are talking about a density of probability represented by an integral (check the definition in the lecture notes). If you compute such integral between two equal boundaries, which is what you should do to answer this question, you will find zero.

3. Suppose that 68% of the drill bits have a diameter comprised between 3.88 and 4.24 mm. Averages of five successive drill bits were measured, and 40% of these were observed to lie between 4.06 and A mm. Estimate the value of A (state the assumptions you make and say whether these assumptions are likely to be true for this example).
   Note. You may want to use online calculators for normal distributions to compute the area under the curve or any parameter of a normal distribution.

Assuming a normal distribution of the diameters, the mean diameter should be 4.06 mm and standard deviation should be approximately 0.18 mm since the 68% interval is at $\pm\sigma$.
As we are using successive averaging, if we assume truly random averaging, the mean for the averaged sample should be $\bar{\mu} = 4.06$ mm and its standard deviation given by the formulas for repeated sampling:

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} = \frac{0.18}{\sqrt{5}} \cong 0.08$$

Based on this normal distribution, the interval spanning 40% will be from 4.06 mm to $A \cong 4.16$ mm.
To find this value, we solve the case $P(x \leq A) = 50\% + 40\% = 0.9$ and use the Stat Trek's calculator (don't forget to add the 50% corresponding to the diameters below the average of 4.06 mm!).

19. Process Control Values

We consider the set of data given in **Figure 20** that shows length measurements (in mm) taken on a machined workpiece. The sample size is 5 and the number of samples is 10, thus the total number of parts measured is 50. The quantity $\bar{x}$ is the average of five measurements in each sample.

1. Determine the upper and lower control limits and standard deviation for this population.

All the required information can be found in the lecture notes.
First, we compute the average of the averages $\bar{\bar{x}}$ and the average of the ranges $\bar{R}$:

$$\bar{\bar{x}} = \frac{1125.14}{10} = 112.51\ \mathrm{mm}$$

$$\bar{R} = \frac{26.0}{10} = 2.60\ \mathrm{mm}$$

To compute the upper and lower control limits, we need to have the constant values A_2, D_3 and D_4. In our case, we have a sample size of 5, which gives us 0.557, 0 and 2.115 for these constants. Thus, we obtain the following upper and lower limits:

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 112.51 + 0.557 \cdot 2.60 = 113.96\ \mathrm{mm}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 112.51 - 0.557 \cdot 2.60 = 111.06\ \mathrm{mm}$$

$$UCL_R = D_4\bar{R} = 2.115 \cdot 2.60 = 5.50\ \mathrm{mm}$$

$$LCL_R = D_3\bar{R} = 0 \cdot 2.60 = 0\ \mathrm{mm}$$

To derive an estimation of the standard deviation for the population of machined parts out of these samples, we use the value of the last parameter d_2 and apply corresponding formula:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{2.60}{2.326} \cong 1.12\ \mathrm{mm}$$

2. What are the consequences of setting the lower and upper specifications closer to the average values $\bar{\bar{x}}$ and $\bar{R}$?

In statistical process control, setting the specifications closer to the center of the distribution will cause more sampling points to fall out of the limits, thus increasing the rejection rate.

3. Identify at least five factors that can cause a process to become out of control. You can discuss several processes.

A process can become out of control because of various factors, such as:

i. the gradual deterioration of coolant or lubricant
ii. debris interfering with the manufacturing operation
iii. an increase or decrease in the temperature in a heat-treating operation
iv. a change in the properties of the incoming raw materials
v. a change in the environmental conditions such as temperature, humidity, and air quality

20. Cost of Quality

High-quality polymer tubes are being produced for medical applications in which the target wall thickness is 2.6 mm, a UCL was set to 3.2 mm, and an LCL to 2.2 mm. If the units are defective, they are replaced at a shipping-included cost of 10 €. The current process produces parts with a mean value of 2.6 mm and a standard deviation of 0.2 mm. The current volume is $10'000$ sections of tube per month. An improvement is being considered for the extruder heating system: it would cut the variation in half, but costs $50'000$ € to implement.

1. Is it correct to assume that the target wall thickness with its tolerances is $t = 2.6^{+0.6}_{-0.4}$ mm?

No, this is wrong. UCL and LCL are not related to design tolerances because they reflect the natural variation of a process, not the specification limits of a product. UCL and LCL are calculated from process data (usually $\pm 3\sigma$ from the mean) and are used to monitor whether a process is stable and in control. Design tolerances, on the other hand, are defined by engineering requirements and specify the acceptable range of variation in a part's dimensions or characteristics.

2. Determine the Taguchi loss function and the payback period for the investment.

The quantities involved are $UCL = 3.2$ mm, $LCL = 2.2$ mm, $T = 2.6$ mm (from design), $\sigma = 0.2$ mm and $Y = 2.6$ mm (from production). Replacement cost is $R = 10$ €. The quantity k is:

$$k = \frac{R}{(LCL - T)^2} = \frac{10}{(2.2 - 2.6)^2} \cong 63\ € \cdot \mathrm{mm}^{-2}$$

Note. As there is a difference between LCL and UCL, one uses the strongest specification to consider the worst-case scenario.

The Loss cost before improvement:

$$Loss = k[(Y - T)^2 + \sigma^2] = 63 \cdot [(2.6 - 2.6)^2 + 0.2^2] = 2.5\ €/\mathrm{tube}$$

The Loss cost after improvement where the variation is cut in half (that is, $\sigma = 0.1$ mm):

$$Loss = 63 \cdot [(2.6 - 2.6)^2 + 0.1^2] \cong 0.6\ €/\mathrm{tube}$$

So the savings after improvement are $2.5 - 0.6 = 1.9$ € per tube, that is, $19'000$ € per month.
The payback period for the investment is $50'000/19'000 = 2.6$ months.

3. Discuss the advantages and disadvantages of the Taguchi method.

The Taguchi method offers significant advantages in improving product quality by emphasizing a performance characteristic value close to the target, rather than simply within acceptable specification limits. This approach leads to better overall quality. Taguchi's method for experimental design is user-friendly and versatile, making it a powerful tool in engineering. It can efficiently narrow the focus of research projects and help identify issues in manufacturing processes using existing data. The method enables the analysis of numerous parameters without requiring excessive experimentation.

However, the Taguchi method has some notable drawbacks. The results it produces are relative, making it difficult to determine which parameter has the most significant impact on performance. It also struggles to account for interactions between parameters, which has been a point of criticism in the literature. Another limitation is that Taguchi methods are offline, making them unsuitable for processes that are dynamically changing, such as in simulation studies. Additionally, these methods are most effective in the early stages of process development when designing for quality. Once design variables are established, continuing with experimental design may become less cost-effective.

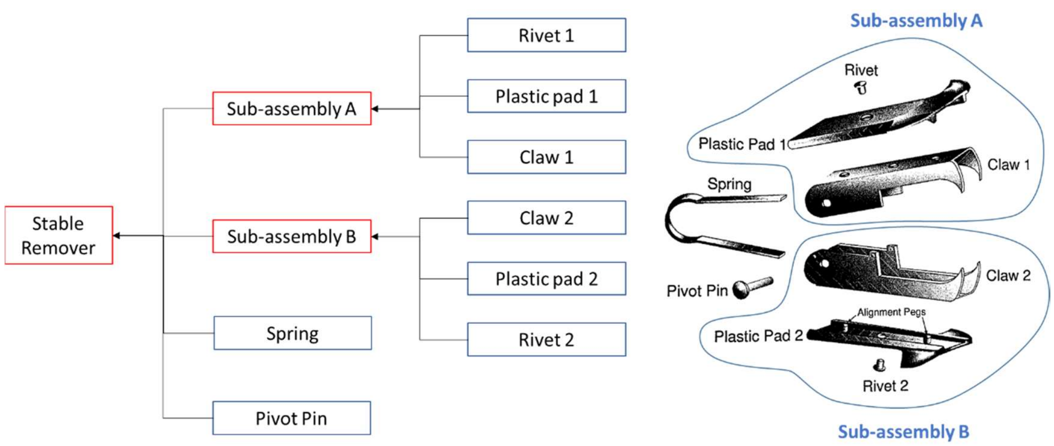
21. Assembly Cost Analysis: The Staple Remover

1. Generally speaking, how would you define what is a sub-assembly?

A sub-assembly is a group of parts that can be mounted separately from the rest of the object, before being assembled to this object.

2. As a first step, define sub-assemblies and make a graph of the assembly.

The graph is shown in the figure below. The first task is to identify sub-assemblies. For each sub-assembly, we notice that there is one main plane of assembly. The penalties of orientation and insertion for each part are evaluated using the methodology described in the lecture notes.



Assembly graph. Two sub-assemblies can be defined and mounted separately.

3. Using your graph, analyze the manual assembly operations and make a cost estimate of it. Assume a labour cost of $C_L = 0.02\ \mathrm{CHF} \cdot \mathrm{s}^{-1}$. Use the Excel table available on Moodle!

Using an Excel table and evaluating the handling and fitting operations for each part (based on the graph in the lectures), an indicative cost of assembly for the stapler remover can be evaluated:

Assembly cost analysis table							Stapler remover															
Labor rate		0.020	CHF/s				71.2	CHF/h														
Components assembly details				Handling operation analysis (H)						Fitting operation analysis (F)												Cost Assembly (in CHF)
Part ref.	Sub-assembly ref	Part description / sub-assembly desc.	Assembly process	Ah	Po1	Po2	Σ Po	Pg	Total Handing	Af	Pf1	Pf2	Pf3	Pf4	Pf5	Pf6	Σ Pf	Pa	Total Fitting	Total (F+H)		
1		Rivet 1	Hand./Fit.	1.5	0.1	0	0.1	0	1.6	2.5	0	0	0	0	0	0	0	0	2.5	4.1	fr.	0.08
2		Plastic pad 1	Hand./Fit.	1	0.1	0.1	0.2	0	1.2	1	0	0	0	0	0	0	0	0	1	2.2	fr.	0.04
3		Claw 1	Hand./Fit.	1			0	0	1	1	0	0	0	0	0.1	0	0.1	0	1.1	2.1	fr.	0.04
4		Rive6 2	Hand./Fit.	1.5	0.1	0	0.1	0	1.6	2.5	0	0	0	0	0	0	0	0	2.5	4.1	fr.	0.08
5		Plastic pad 2	Hand./Fit.	1	0.1	0.1	0.2	0	1.2	1	0	0	0	0	0	0	0	0	1	2.2	fr.	0.04
6		Claw 2	Hand./Fit.	1	0.1	0.1	0.2	0	1.2	1	0	0	0	0	0.1	0	0.1	0	1.1	2.3	fr.	0.05
	A	#1+#2+#1	Hand./Fit.	1	0.1	0.1	0.2	0	1.2	1	0	0	0	0	0	0	0	0	1	2.2	fr.	0.04
	B	#4+#5+#6	Hand./Fit.	1	0.1	0.1	0.2	0	1.2	1	0.1	0	1	0	0.3	0.1	1.5	0	2.5	3.7	fr.	0.07
7		Spring	Hand./Fit.	1	0.1	0	0.1	0	1.1	1	0.1	0	0.2	0.2	0	0	0.5	0	1.5	2.6	fr.	0.05
8		Pivot pin	Hand./Fit.	1	0.1	0	0.1	0	1.1	1	0	0	0	0	0	0	0	3	4	5.1	fr.	0.10
																			Total cost		fr.	0.61

You can assume that your answer is correct if it falls within $\pm 1\%$ around our numerical answer.